Revised: March 2010

May Growth Lead to Higher Deprivation Despite Higher Satisfaction?

Quentin Wodon and Shlomo Yitzhaki[1]

World Bank and Israeli Central Bureau of Statistics

Abstract

In a relative deprivation framework, unless inequality is reduced, growth is associated with both higher satisfaction and higher deprivation. This may help explain the discontent with growth despite its benefits. Knowledge of the population's mean income and the Lorenz curve is all that is needed to assess the satisfaction and deprivation of each individual. This is because for both the population as a whole and each individual, satisfaction and deprivation add up to the mean income. The first part of the paper analyzes the effect of growth on relative deprivation, while the second part presents the adjustments needed to include relative deprivation as a target in models intended to derive policy implications.

Key words: growth, inequality, satisfaction, deprivation
JEL categories: D63, O15

Corresponding author:
Address: Shlomo Yitzhaki
Dept. of Economics
Hebrew University
Jerusalem, 91905
Israel
Email: yitzhaki@cbs.gov.il

[1] The views expressed here are those of the authors and need not reflect those of The World Bank nor the Israeli Central Bureau of Statistics.

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1. Introduction

This paper deals with a paradox. Many countries have experienced substantial economic growth in the last two decades (even if this has been dampened recently by the current global economic crisis), and yet there is widespread discontent with the distribution of the benefits from growth. This may reflect a feeling of economic insecurity due to trade openness and liberalization. But it may also reflect feelings of deprivation despite higher standards of living. More precisely, while an individual's well-being or satisfaction depends on what the individual actually consumes, deprivation depends on what the individual is not able to consume, but would like to consume because he/she sees others consuming it. If growth leads to a larger pie and more goods to be desired, it can also lead to a higher feeling of deprivation. In order for this feeling of deprivation to be reduced, growth may have to be accompanied by lower inequality.

The concept of relative deprivation was put forward by Stouffer et al. (1949) in their monograph *The American Soldier: Adjustment During Army Life*, and the classic reference is Runciman (1966),[2] but Rainwater (1974) suggests that the concept goes at least back to Marx, who wrote: "*Our desires and pleasures spring from society, we measure them therefore, by society and not by the objects which serve for their satisfaction. Because they are of a social nature they are of a relative nature* (quoted by Pedersen, 2004, p-39)." Note that the relativity of the concept comes from the reference to the society. Keeping an individual's consumption unchanged, while changing the consumption of other members of the society, may affect the individual's satisfaction.[3] Among economists, the concept has not been used extensively[4], in

[2] Runciman (1966) wrote his book to explain another paradox – that according to his observations, in Britain, the lower the observed inequality, the higher the feelings of deprivation are.

[3] The group from which the individual derives his aspirations is referred to as the reference group. In the general case, an individual may have several reference groups, and the reference groups of different individuals may differ and change over time. Those extensions are beyond the scope of this paper. Yitzhaki (1982, 2009) offer some analysis on the implications of reference groups.

[4] As noted by Pedersen (2004), a handful of economists have considered deprivation in applied work, including Layard (1980) on the role of status ranking in motivation, and members of the so-called Leiden school for work on poverty and well-being (van Praag 1971; Kapteyn et al 1980; Kapteyn and Wansbeek 1982; van de Stadt et al. 1985). See also the axiomatic definitions of relative deprivation proposed by Podder (1996). There are two fields in which relative deprivation is extensively used: Health and mortality (Eibner and Evans, 2001; Deaton, 2001), and migration (Quinn, 2001; Stark, 1984; Stark and Taylor 1989; Stark and Wang, 2000; Stark and Yitzhaki, 1988).

part because economists typically assume that the satisfaction provided to any one individual by his/her income does not depend on the income of others, or more precisely, on his/her rank in the overall distribution of income. This assumption is relaxed in Yitzhaki's (1979, 1982) formalization of Runciman's theory of relative deprivation, which leads to a social welfare function equal to the product of the mean income in the population as a whole times one minus the (extended) Gini index of inequality.

There is empirical evidence that individuals do care about relative consumption and deprivation (see for example Alpizar et al., 2005). There is also a debate regarding the link between growth and various measures of subjective well-being. Easterlin (1974, 1991, 1995, 2005) suggested a lack of relationship between economic development and average levels of happiness, but new evidence provided by Stevenson and Wolfers (2008) suggests that higher GDP per capita level may lead to higher subjective feelings of well-being. Part of the complexity in these relationships between growth and well-being stems from the fact that many factors affect subjective feelings. That is, the relationships between growth and feelings of deprivation, satisfaction and happiness is not straightforward, and these various concepts may refer in the mind of authors to quite different things. In this paper, we focus on the links between growth, deprivation and satisfaction within Yitzhaki's relative deprivation framework. This of course does not mean that other factors do not also affect feelings of deprivation, satisfaction and more generally happiness, as pointed out by Dean (2007) among others.

While other authors (e.g., Berrebi and Silber, 1985; Bishop, Chakraborti and Thistle, 1991, and Sen, 1973) have used the same social welfare function, Yitzhaki relies on this function specifically in the context of relative deprivation, as opposed, for example, to the argument that individuals may compare themselves to others when evaluating their level of well-being (Duclos, 1998, and Hey and Lambert, 1979). It may well be true that individuals compare themselves to others,[5] and the Gini index itself can be written as a sum of such pairwise comparisons of income, but the logic in Runciman's argument is different. It relies on the idea that scarcity determines the value attached to each unit of income, and in Yitzhaki's formalization of this

[5] Duesenberry (1951), Frank (1985), and Kapteyn et al. (1980) present evidence on the dependency of preferences on others. For a survey of the issues involved, see Weiss and Fershtman (1998).

idea, on a given measure of scarcity. Ebert and Moyes (2000) present axiomatic derivation for the relative deprivation function.[6]

The idea that individuals value goods (in order to assess their satisfaction or deprivation) in relationship to their rank in the overall population as a measure of scarcity has been suggested in other settings. Within a firm, the value of a position depends on how high the position is in the overall distribution of positions (Stark and Yitzhaki, 1988). The smaller the number of Vice Presidents, the higher the value of being a Vice President.

Another imperfect, yet possibly revealing analogy is that of the decathlon, an athletic two-day event with ten different disciplines (100 meters sprint, long jump, shotput, high jump, and 400 meters in that order on the first day, and 110 meters hurdle, discus, pole vaults, javelin, and 1500 meters on the second day). A scoring table is used to award points for performance in each discipline, and the winner is the athlete with the highest total score after the ten events. To draw the analogy with a market economy, the ten disciplines can be considered as commodities, and the scoring system as prices. The utility function of each athlete may be defined over the physical units of achievements in every field. The first stage in constructing the analogy, can be referred to as the micro-economic problem: each athlete allocates his/her practicing time in order to maximize his/her utility subject to a time constraint and the scoring structure (prices). A proper solution (in a competitive environment) is to allocate time so that, for each athlete, the marginal cost of achieving each additional point is equal for the various events.[7] The allocation of time and effort may also be interpreted as if each athlete were maximizing his/her points (income) subject to the time constraint. The second stage is the general equilibrium process which determines the prices. To reach an "equilibrium", the scoring structure is adjusted by a committee to avoid that any one discipline overshadows all others (in our analogy, this is akin to the demand and supply mechanisms in markets).

The result of applying the prices (scores) to individual achievements in each field to get the total score of the individual athlete is that we end up with points of equal value from the

[6] There have also been a number of extensions of and alternatives to the approach proposed by Yitzhaki to analyze relative deprivation. For example, Verme and Izem (2008) suggest an alternative index of relative deprivation that allows for the selection of the reference group and imperfect information, with an application to US wage data. See also Hopkins (2008) for a survey of different theoretical models of relative concerns and their link to inequality.

[7] A monopolistic behavior implies that an athlete takes into account that his achievements may affect the scoring system. Note, however, that the arguments raised continue to hold under monopolistic behavior.

point of view of production (given the pricing system, each point requires an equal marginal cost or effort to produce). However, an athlete's satisfaction depends not only on his/her total score, but also on the achievement of others; in other words, it depends on his/her ranking. We refer to this process of evaluation as the social evaluation, and relative deprivation theory is applied at this stage. We will show that given the distribution of abilities, and given our formalization of relative deprivation theory, the social evaluation for each athlete can be summarized by the number of points achieved and the rank of the athlete. We argue that there is no envy nor altruism, except the need to be able to evaluate the achievement of the various athletes. We show that if there is an overall inequality-neutral improvement in scores, there is higher satisfaction (all athletes have improved their personal record), but there is also higher deprivation because the distance separating the athletes from those at the very top has increased. However, if there is a constant increase in points that is equal among all athletes, then satisfaction increases while deprivation stays at the same level.

The same idea can be applied to growth. In a relative deprivation framework, unless inequality is reduced, growth is associated with both higher satisfaction and higher deprivation, which may help explain the discontent with growth despite its benefits. This is in turn related to a distinction by Kolm (1976) between "rightist" and "leftist" inequality measures. A rightist inequality measure does not change if a proportionate amount of income is added or subtracted from all incomes. This is of course a property of most traditional measures of inequality, including the (extended) Gini index. By contrast, a leftist inequality measure does not change if an absolute amount of income is added or subtracted from all incomes. We will show that the concept of deprivation, as used here, is a leftist approach.

In what follows, we use Yitzhaki's formalization of deprivation theory to analyze the impact of growth and changes in inequality on satisfaction and deprivation. The main conclusion is very simple. Growth will often increase both satisfaction and deprivation in a society. The fact that growth increases satisfaction is trivial. The fact that it also increases deprivation is less clear. It occurs because growth also increases the scope of commodities to be desired. Therefore, unless growth is accompanied by a reduction in inequality, it is not going to decrease the feelings of deprivation in countries that experience it, and therefore we should expect dissatisfaction from growth. This may help explain part of the discontent in many countries, even in the countries that have done relatively well over time. The paper provides also

a number of new results, as well as an intuitive graphical interpretation of the concepts of satisfaction and deprivation. The most important new result is that the knowledge of the population's mean income and the Lorenz curve is all that is needed to assess the satisfaction and deprivation of each individual. This is because for both the population as a whole and each individual, satisfaction and deprivation add up to the mean income.

The structure of the paper is the following: Section 2 presents the main argument, while section 3 discusses the implications for policies that adhere to the concept of growth with no deprivation. Section 4 concludes.

2. Growth, Satisfaction, and Deprivation

In textbook economics, the social welfare function is assumed to be symmetric and additive. Symmetry implies that two individuals with the same income have the same level of welfare. Additivity implies that welfare for society as a whole is the sum (possibly adjusted for population) of individuals' welfare. Denoting social welfare by W, individual income by I_i, prices by p, and the individuals' indirect utility function by w (assuming that two individuals with the same income have the same social evaluation of the indirect utility function), an additive social welfare function can be written as:

$$W = \sum_{i=1}^{N} w(I_i, p) \qquad (1)$$

The social welfare function is intended to serve as a *decision function.* That is, it describes how society should make its decisions. On the other hand relative deprivation is intended to be descriptive in nature, an early version of the empirical literature about happiness and satisfaction from economic well-being. That is, it is intended to describe individuals' feelings. [8] It includes three dimensions: income, prestige and power. Like the topic in this paper, Runciman developed the theory in order to explain a paradox: the rise in resentment and deprivation in periods of decrease in inequality. In this paper we deal with income only. According to Runciman (1966:10): "*[a person] is relatively deprived of X when (i) he does not have X, (ii) he sees some other person or persons, which may include himself at some previous or expected time, as having X (whether or not that is or will be in fact the case), (iii) he wants X, and (iv) he sees it as feasible that he should have X.*" In formalizing Runciman's discussion,

[8] See Clark and Oswald (1996), Eeasterlin (1974, 2005) and Senik (2007) .

Yitzhaki (1979, 1982) suggests that each unit of income (say, a dollar), represents a basket of commodities that individuals can buy (a unit of income should be interpreted as normalizing the different commodities to units of equal cost to produce; we will elaborate on this point later.) Each dollar represents a bundle of commodities with equal marginal alternative cost, but the social value for the individual (the marginal utility or the social evaluation of the marginal utility) associated with the first unit of income (dollar) is higher than that associated with the second, which in turn is higher than that associated with the third, etc. While each unit of income can be considered as X in Runciman's quote above, the value that individuals attach to each X depends on the number of individuals they see having X. There is no envy, nor altruism it is simply a way of evaluating what one has.

Following Runciman, Yitzhaki suggests to measure deprivation (and satisfaction) by the share of the population, which has X.[9] As shown in the appendix, it is possible to generalize the function by allowing deprivation to be a decreasing function of the share of the population which have X. This, then, uses the extended Gini as opposed to the standard Gini presented below. In other words, all the results presented here can be generalized to allow flexible (social) preferences. Since the gain from such an extension is overshadowed by the complexity, this extension is relegated to the appendix.

The dependency of the social valuation on scarcity may be interpreted as an extension of the law of diminishing marginal utility, which says that ceteris paribus, the more of commodity X one has, the lower its marginal utility. Applying the same rule to units of income, the more people one sees as not having the unit, the less deprived one feels from not having it. Alternatively, the more people one sees having it, i.e., the less people are deprived of it, the more important for a person to have the unit and hence the more the person feels deprived from not having it. Denoting by $F(I_i)$ the position of individual i in the cumulative distribution of income, this leads to a social evaluation of the marginal utility of income equal to $1-F(I_i)$, because this term represents the share of people in the society who have the I-th unit of income. The satisfaction of the individual is defined as the sum of the satisfaction derived from each unit of income he possesses while his deprivation is the sum of deprivations from all units of income

[9] We assume that there is one reference group, identical to all members of the society, and it is the society as a whole.

that he does not possess. This yields the following values for individual satisfaction $s(I_i)$ and deprivation $d(I_i)$:

$$s(I_i) = \int_0^{I_i} [1 - F(z)]dz \qquad \text{and} \qquad d(I_i) = \int_{I_i}^{I_B} [1 - F(z)]dz \,, \tag{2}$$

where I_B is the maximum income observed in the society. That is, the social value of the income is the sum of the social values of the units of income one possesses, while deprivation is the sum of the feelings of deprivation on all units of incomes one is deprived of. For each individual, satisfaction and deprivation add up to the population's mean income. To see this note that:

$$s(I_i) + d(I_i) = \int_0^{I_B} [1 - F(z)]dz \,. \tag{3}$$

Integrating by parts the right hand side term, with u=1-F(z), u'=-f(z), v'=1, v=z, leads to:

$$\int_0^{I_B} [(1 - F(z)]dz = [1 - F(z)]z|_0^{I_B} + \int_0^{I_B} zf(z)dz = \mu \quad . \tag{4}$$

In Equation (2) deprivation is defined on the units of income that the individual does not have (but sees other people having). Satisfaction is defined on the units of income the individual possesses. In this sense, deprivation is a mirror image of satisfaction.

It is shown in the Appendix, that the satisfaction of individual i can also be written as:

$$s(I_i) = \mu[(1 - F_i)L'(F_i) + L(F_i)] \,, \tag{5}$$

where F_i is the rank of individual i in the society, L() is the Lorenz curve, and L'() is the derivative of the Lorenz curve (the derivative of the generalized Lorenz curve is μ L'().) Note that although (5) suggests that satisfaction is a decreasing function of the rank of the individual in the society, a full derivative would show that satisfaction is an increasing function of rank. Note also that while the satisfaction of each individual is a function of his/her income, (5) shows that given the Lorenz curve, one can find the satisfaction of each individual by knowing only the mean income in the society and the rank of the individual in the distribution of income.

The deprivation of the individual is:

$$d(F_i) = \mu - w(F_i) = \mu[(1 - L(F_i) - (1 - F_i)L'(F_i)] \,, \tag{6}$$

which implies again that given the shape of the Lorenz curve, individual deprivation can be estimated by knowing the rank of the individual, and the mean income in the population.

Figure 1, which provides a graph of the generalized Lorenz curve, shows the connection between the rank of the individual and his/her satisfaction and deprivation.[10] On the vertical axis on the right, the distance from the horizontal axis to A is equal to $\mu L(F_i)$, and the distance from A to B is equal to $\mu(1-F_i)L'(F_i)$. Hence individual satisfaction is measured by the height of B. Since for each individual satisfaction and deprivation add up to the population's mean income, the distance from B to C is the measure of individual deprivation. At the aggregate level, the area between the diagonal 0C and the generalized Lorenz curve is equal to half the population's deprivation, namely $\mu G/2$. The area below the generalized Lorenz curve is equal to half the population's satisfaction, namely $\mu(1-G)/2$. The fact that for a given mean, a higher level of inequality leads to lower satisfaction and higher deprivation is thus easy to visualize. It can also be shown that the higher the rank of any individual, the lower his/her deprivation is, and the higher his/her satisfaction is as well. The proof is based on the convexity of the Lorenz curve.[11]

Aggregating satisfaction and deprivation in society as a whole yields:

$$S = \int_0^{I_B} s(z) f(z) dz \text{ and } D = \int_0^{I_B} d(z) f(z) dz \quad . \qquad (7)$$

Denoting by G the Gini index of inequality, and by μ the mean income in the population, Yitzhaki (1979) proves that (5) is equivalent to:

$$S = \mu(1-G) \text{ and } D = \mu G \quad . \qquad (8)$$

Finally, it is worth emphasizing that one could use more complicated measures of the deprivation resulting from the scarcity of the I-th unit of income, such as $(1-F(I))^v$, where $v > 1$ is a parameter determined by the investigator, to get the same qualitative results. See the Appendix.

A key implication of the above is that in principle, knowing whether mean income and inequality have increased or decreased (without knowing the magnitude of the changes in each) is insufficient to know the direction of the change in satisfaction and deprivation. However, because inequality in per capita income tends to change at a much slower pace than mean per capita income, it is likely that the impact of growth on satisfaction and deprivation will be larger than the impact of changes in inequality. This leads to the paradox alluded to in the introduction.

[10] The concept of rank is usually associated with positional goods (e.g., Frank, 1985). Note, however, that we did not assume here that the units of income are positional goods, and we have derived the rank as a summary statistics that describes the satisfaction of the individual.

[11] A sketch of the proof: Let $F_1 < F_2$ be two ranks, and let $L(F_1) < L(F_2)$ be the associated points on the Lorenz curve. Because of the convexity of the Lorenz curve, $L(F_2) > L(F_1) + (F_2 - F_1) L'(F_1)$ and $[L'(F_1) - L'(F_2)](1 - F_2) < 0$.

If inequality remains unchanged, while growth will lead to higher satisfaction, it will also lead to higher feelings of deprivation, in the sense that what individuals do not have, but wish they had and believe they could have (because others now have it), will increase as well.

It is worth stressing that by assuming the society as the only reference group, we have neglected the "relative" component in relative deprivation theory. The relativity of the concept comes from an endogenous selection of one's reference group. We have not dealt with this component not because we think it is unimportant, but because we have mathematical difficulties in presenting a theory that can also include the endogenous creation of reference groups, a subject that requires other components of the theory, like power and group identity. Moreover, one can have several reference groups and the fact the A is in the reference group of B does not mean that B is in the reference group of A. Further research is needed to deal with those issues.[12]

3. Discussion on policy recommendations.

Having described relative deprivation as an alternative to the Bergson-type social welfare function, we now raise the question as to whether policy recommendations would be affected when adopting a relative deprivation framework.[13] This is a difficult question to answer in a precise way, given that the implications for policy are likely to depend on the type of policy considered. At a very general level, we can assume that it is likely that the relative deprivation framework would put more emphasis on the reduction of inequality than a traditional Bergson-type welfare function. But in practice, whenever a current policy is far away from the optimum, it may well be that there might not be major differences between recommendations stemming from a Bergson-type social welfare function or from a relative deprivation framework.

Said differently, the choice of the welfare function will probably matter most when dealing with fine tuning of policy recommendations. In addition, clearly one potential source of major deviation between recommendations derived from relative deprivation theory as opposed to the Bergson-type social welfare function may arise from the "relative" part of deprivation theory, which is caused by viewing the society as composed of several reference groups. Yet we have not considered this here to any substantial extent.

[12] Frick *et. al.* (2006) offer a decomposition of the Gini coefficient to sub-populations that may be useful in analyzing the effect of different reference groups on deprivation. However, further research is needed.
[13] The need for a new kind of research in this area is stressed by Dean (2007).

In any case, in order to give a conceptual illustration, to see the difference between the two approaches consider a typical problem of optimal taxation with heterogeneous taxpayers, as presented in Diamond (1975). In this problem, the government seeks to optimize the following function:

$$\text{Max} \quad W(v_1(I_1),\ldots,v_H(I_H),P_1,...,P_N) \tag{9}$$

$$t_1,...,t_k$$

$$\text{s.t.} \quad R_0 = \sum_{i=1}^{K} t_i X_i \quad . \tag{10}$$

Here, W is the Bergson-type social welfare function, v_h is the indirect utility of individual h, $P_1,..,P_N$ are the prices that the consumers face, t_i is the tax rate on taxable commodity i, X_i is the aggregate quantity consumed of commodity i, and K is the number of tax commodities. On the other hand, substituting the social welfare function by a function representing relative deprivation means that we now seek to maximize:

$$\text{Max} \quad \mu\,(1\text{-}G) \tag{11}$$

$$t_1,...,t_k$$

$$\text{s.t.} \quad R_0 = \sum_{i=1}^{K} t_i X_i$$

where μ is mean income, G is Gini coefficient of the after-tax income.

Solving either (9) or (11) requires many (often unsubstantiated) assumptions concerning the tastes of the individuals in the society and the exact specification of the social welfare function. The problem is simplified, and a great deal of information saved, if we restrict ourselves to finding the recommended direction for tax reform. This is because in the analysis of marginal tax reform we do not seek to find the optimal tax rates but only the direction on the margin in which the tax system should move. The main disadvantage is that instead of finding out the optimal taxes we restrict ourselves to finding out which tax rates to increase or decrease, without specifying the exact required change. The main advantage, on the other hand, is that we do not have to assume that the government acts in an optimal way, only that the individuals do.

This kind of exercise is suggested in Mayshar and Yitzhaki (1995). It is assumed that the only information known on the social welfare function is that the social evaluation of the marginal utility of income is positive and declining with income. This approach is referred to as second-degree welfare dominance. Yitzhaki (1982) showed that (11) forms a necessary condition for second-degree welfare dominance, while Shorrocks (1983), generalizing the seminal findings

in Atkinson (1970), showed that a necessary and sufficient condition for second-degree welfare dominance is that the generalized Lorenz curves of the after tax income distributions do not intersect. Following this kind of mild assumptions, one can find out major deviations from optimal taxation, whenever distributional issues are ignored. Yitzhaki and Mayshar (1995) found that taxation of alcohol products and gasoline in Britain deviated from the optimum. Yitzhaki and Lewis (1996) found that the policy recommended by the World Bank was not welfare improving because it ignored consumption patterns of the poor in Indonesia, while Yitzhaki (1990) found that the urban bias caused subsidies for basic food items in Egypt to deviate from the optimum.

Makdissi and Wodon (2002) as well as Duclos, Makdissi and Wodon (2008) have expanded the analysis to cover poverty separately and to include first degree welfare dominance[14]. Bibi and Duclos (2007) have applied this extended methodology to Tunisia and found a large deviation from optimal policy while Lundin (2001) expanded the methodology to handle externalities. However, under those mild assumptions there is no difference between Yaari's (1987) dual theory, welfare dominance, or relative deprivation.

The drawback of the welfare dominance approach is that it offers only a partial order of possible tax reforms, and in many interesting cases the researcher is unable to reach a conclusion. In many developed societies those are the really interesting issues because one does not expect large deviations from the optimum, and the question of interest is what are the trade-offs that should be considered in the planning of the tax reforms. Technically, no conclusion can be reached whenever the Generalized Lorenz curves calculated before and after the reform intersect. This is the typical case. To fully evaluate the alternative tax reforms, one needs a complete order over them. In this case, there is no alternative but to use (11) or some alternative fully specified welfare function in order to evaluate the reform.

Stemming from (11), the literature analyzing tax reforms using the change in the Gini coefficient calculated over after-tax income is surveyed in Wodon and Yitzhaki (2002).[15] This analysis has been performed for many countries, and it relies on one key parameter, the Gini income elasticity (GIE). The GIE is the income elasticity of the tax base with respect to the adjusted after-tax income. The only difference between it and the regular estimation of the Engel

[14] Under first degree welfare dominance there is only one assumption, that the social evaluation of the marginal utility of income is positive.
[15] The analysis of how to derive policy recommendation concerning growth is on pp-95-98.

curve of the tax base is that a Gini regression (Olkin and Yitzhaki, 1992) is performed. Unlike the OLS regression which tends to stress the behavior of the rich even when one is interested in poverty, the Gini regression is designed to take into account the social views of the researcher (Yitzhaki, 1996). The advantage of using this methodology in analyzing tax reforms is that $\mu(1-G)$ forms a necessary condition for welfare, relative deprivation, and Yaari's (19) dominance criteria so that it is impossible to prove that no accepted social welfare function justifies the reform. On the other hand, using Ordinary Least squares regression to estimate the Engel curves needed for the evaluation of the reform may contradict all accepted social welfare function (Yitzhaki, 1996). This may be important whenever the specification of the underlying model is not correct. However, unlike the analysis of welfare dominance, the above literature ignores the cost of taxation.

4. Conclusion

Our main objective in this paper has been to provide a better understanding of the nature and properties of the theory of relative deprivation, as formalized by Yitzhaki (e.g., 1979, 1982). In a relative deprivation framework, unless inequality is reduced growth will be associated with both higher satisfaction and higher deprivation, so that there may be some discontent with the growth process even though it does clearly provide benefits. The theory of relative deprivation may help in explaining the apparently widespread discontent with traditional growth processes which have not led to reductions in inequality. While the framework does not suggest how to achieve both growth and a reduction in inequality, it points to the necessity of further research on this subject since growth alone is probably not good enough.

The paper also provides new results and an intuitive visualization of the concepts of satisfaction and deprivation. We showed that in order to assess an individual's satisfaction and deprivation, we only need to know the population's mean income, the Lorenz curve, and the individual's rank in the distribution of income. This is because the indices representing satisfaction and deprivation add up to the mean income not only for the society as a whole, but also for each individual.

We also discussed briefly the effect of adopting relative deprivation, rather than the Bergson-type social welfare function, for actual policy recommendations. We have suggested that whenever we are dealing with large deviations from an optimum, it may not matter too much which theory is adopted. On the other hand, if the aim is fine tuning the policy, then one can use

relative deprivation as a guide for policy according to an already developed statistical methodology. This statement has to be qualified, however. The essence of relative deprivation is the existence of different reference groups for different individuals. Further research on the formation and changes of reference groups is needed to fully exploit the theory. For a preliminary discussion on this topic, see Yitzhaki (2009) and references therein.

Appendix

This appendix proves the main results of the paper. We will derive the results using the extended Gini. To derive the results presented on the Gini, simply set ν=2, and repeat the steps. Assume that due to scarcity, the value attached to a unit of income is $[1-F(y)]^{\nu-1}$, where $\nu>1$ is a constant and F(y) is the cumulative distribution of income. Note that 1-F(y) is the number of individual who do posses income unit y. The satisfaction of the individual is the sum of the satisfactions from all the units of incomes that he/she possesses, while the deprivation is the sum of the deprivation from all the units that he/she does not have. That is, for an individual with income I_i:

$$s(I_i) = \int_0^{I_i} [1 - F(y)]^{\nu-1} dy \quad \text{and}$$

$$d(I_i) = \int_{I_i}^{\infty} [1 - F(y)]^{\nu-1} dy .$$

Using integration by part with $u=[1-F(y)]^{\nu-1}$, $u'=-(\nu-1)[1-F(y)]^{\nu-2} f(y)$, $v'=1$, $v=y$, we get:

$$s(I_i) = [1 - F(I_i)]^{\nu-1} I_i + \int_0^{I_i} [1 - F(y)]^{\nu-2} yf(y)dy .$$

By transformation of the variable $F_i=F(I_i)$, and using $I_i/\mu = L'(F_i) = \partial L/\partial F$ as the derivative of the Lorenz curve, one gets

$$s(I_i) = \mu[(1 - F_i)^{\nu-1} \frac{\partial L}{\partial F} + \nu \int_0^{F_i} (1 - F)^{\nu-2} \frac{\partial L}{\partial F} dF] .$$

This implies that given the Lorenz curve, the mean income in the society and the rank of each individual in the distribution of income fully determine the satisfaction and the deprivation of the individual in the society. Yitzhaki (1983) then shows that summation of the satisfaction over all members of the society leads to:

$$S = \mu(1 - G(\nu)) = \int_0^{\infty} s(I) f(I) dI$$

where G(ν) is the extended Gini. Similarly, Average deprivation in the society is:

$D = \mu\, G(\nu)$.

Acknowledgement: We are grateful to Victoria Roshal and participants of seminars held in the Hebrew, Oxford and Cornell universities for helpful comments.

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Figure 1: The Generalized Lorenz Curve, Satisfaction and Deprivation.