Dollar Cost Averaging

Michael J. Brennan

Feifei Li

Walter N. Torous *

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*Michael Brennan is Emeritus Professor, Feifei Li is a doctoral candidate, and Walter Torous is Professor Finance at the Anderson School, UCLA. Corresponding Address: Brennan: The Anderson Graduate School of Management; University of California, Los Angeles; 110 Westwood Plaza; Los Angeles, CA 90095-1481. We thank the editor and the referees for helpful comments on an earlier draft. The paper was presented at a conference organized by La Fundacin Ramn Areces in Madrid, 2005.

Abstract

Dollar Cost Averaging is a strategy for purchasing equity securities that is widely recommended by professional investment advisors and commentators, but which has been virtually ignored by academic theorists and textbook writers. In this paper we explore whether the strategy is but another instance of irrational behavior by individual investors, or whether it is an investment heuristic that has survival value in an environment in which security prices exhibit mean reversion behavior that has only belatedly been recognized by academic theorists. Our evidence supports the view that the individual investors who follow this strategy in purchasing individual stocks to add to an existing portfolio are better off than if they followed the 'rational' strategies traditionally recommended by academics.

1 Introduction

Practical or tacit knowledge typically precedes scientific knowledge. Crops were rotated long before the chemical basis of the practice was understood. Men learned to fly before aeronautics was well understood. Extracts of willow were described by Hippocrates as a pain remedy well before Bayer first synthesized aspirin. Acupuncture was in use for millennia before Nixon's 1972 trip to China made the practice scientifically acceptable in the West. And these are only a few examples of the practical wisdom that is embodied in pre-scientific prescriptions. Therefore, given the relatively brief period of scientific study of financial markets, and the controversy that surrounds the interpretation of many of the findings, it is not surprising to find that a good deal of financial practice is still governed by pre-scientific heuristics or maxims. Examples of such maxims include, 'buy on the rumour, sell on the news', 'cut your losses and let your profits run', and the '60-40' and '110 minus your age' rules for asset allocation. Just as we would expect the practices of folk medicine that have survived to possess some curative properties, or at worst to be harmless, so we might expect these investment maxims also to have some value or, at worst, to be harmless, unless they conflict with strongly supported scientific evidence. In this paper we are concerned with the 'dollar cost averaging' (DCA) heuristic.

Financial markets are somewhat different from agricultural crops, flying machines and human bodies as objects of scientific study, since the properties of markets are affected by the beliefs and practices of investors in the markets, and these beliefs and practices are influenced in turn by scientific studies of the markets.[1] This feedback between human decisions, the properties of markets, and the resulting profitability of the decisions has led some scholars to study decision rules for participants in financial markets that have survival value against other decision rules.[2] This is a very different approach from the standard rational expectations approach in which all investors[3] are assumed to have full information about the structure of the economy and about each other's beliefs.

[1] Brennan (2004) argues for example that the efficient markets hypothesis and popular knowledge of the historic equity premium led investors to pay excessive prices for stocks in the 1990's on the basis that stocks beat bonds in the 'long run'.

[2] Blume and Easley (1992), Routledge (2001), Lo(2002), Hens *et al.* (2003).

[3] With the possible exception of some 'noise traders'

Behavioral finance is yet a third approach that has developed of late in reaction to the inability of standard rational expectations models to explain a number of stylized facts about financial markets.[4] Proponents of this paradigm believe, not only that a significant proportion of investors are *irrational*, but also that their irrationality has important implications for market prices; however, the paradigm has not been exploited to derive formal prescriptions for investors operating in these markets.[5]

In contrast to the behavioral position, this paper takes seriously the possibility that the practical maxims or heuristics that we have inherited are not manifestations of investor irrationality, but are actually helpful for investors who operate in markets that do not conform to the simple descriptions of standard asset pricing models, even if the reasons for the failure of these models remain obscure. We examine the value of the dollar cost averaging heuristic by randomly choosing investments dates and securities from the CRSP database over the period 1926-2003 and comparing the certainty equivalent wealth levels yielded by lump sum and dollar cost averaging investment strategies for an investor with iso-elastic utility.

We find, first, that, for an investor who is purchasing a diversified investment portfolio of common stocks represented by the CRSP value-weighted or equal-weighted indices, the DCA strategy, carried out over implementation periods of from one to six years, outperforms the lump sum investment strategy for all except the most risk tolerant investors.[6] This seems to be due to the lower level of risk associated with the DCA strategy since the expected returns are, not surprisingly, higher for the lump sum strategy. However, we also find that both strategies are outperformed by a simple strategy of investing 50% of wealth in stocks and 50% in cash, and rebalancing monthly to maintain the proportions.[7] As it turns out, the 50:50 strategy has almost the same expected return as the DCA strategy, but lower risk. Therefore we find only mixed support for a DCA strategy when applied to the purchase of a diversified portfolio of common stocks.

[4] See also the theory of 'rational beliefs' of Kurz (1994,a,b).

[5] However, a number of investment funds claim to be operated on behavioral principles. For example Fuller and Thaler Asset Management utilizes a 'bottom-up investment approach that combines fundamental research with insights from behavioral finance...'. (*http://www.fullerthaler.com/*

[6] Of course, both strategies are outperformed by the 'optimal' investment strategy which bases the allocation to equities on the realized sample moments of the equity returns.

[7] Brennan and Torous (1999) show that rebalancing strategies are slightly more efficient than buy and hold strategies.

Secondly, when the DCA strategies are compared with the lump sum investment strategy for the purchase of a single stock which is the investor's sole investment, the DCA strategy overwhelmingly dominates the lump sum strategy. The reason for this is the lower risk of the DCA strategy, and it is worth noting that for all except the most risk tolerant investors, the certainty equivalent wealth per dollar of initial investment is less than one - *investment in a single stock portfolio is, in general, not an economic proposition* - even when it is done by DCA.

Thirdly, given the inefficiency of a single stock portfolio, it is natural to consider the merits of DCA when applied to the purchase of an additional stock by an investor who already holds a portfolio of common stocks. When we analyze the case in which the investor is assumed to hold the (value- or equal-weighted) market portfolio and to purchase a marginal amount of a randomly chosen additional stock, the DCA strategies dominate the lump-sum purchase strategy in all cases for investment horizons up to 48 months. If one were to choose an investment strategy for 'everyman' assuming that he holds the (value- or equal-weighted) market portfolio, then a DCA strategy executed over 36-48 months would be the preferred strategy. This result is re-inforced when the investor is assumed to hold initially a diversified portfolio of common stocks *that is optimally levered* given his level of risk aversion. Now DCA dominates the lump sum investment strategy for an investor who holds the value weighted market portfolio for all levels of risk aversion and implementation periods up to 60 months, and the advantage of DCA is greatest for the 36 month DCA strategy where it dominates the lump-sum strategy by a margin of 3-6% depending on the level of risk aversion and the random method used to select securities. When the initial portfolio is the optimally levered equal weighted market portfolio, the advantage of DCA disappears or is even reversed.

While, it is hard to determine the precise features of the joint stochastic process of common stock returns that accounts for this success of DCA, simulations reveal that transient departures of common stock prices from their fundamental values in the manner suggested by Poterba and Summers (1988) can give rise to an advantage to DCA which is similar to that we find in the data.

In summary, we find that dollar cost averaging, when applied to the addition of a stock to an existing portfolio, has benefits that have not received attention in the academic literature despite the fact that investment practitioners have tended to advocate the strategy for many years. It appears to be a case in which practical wisdom has discovered what theoretical knowledge has yet to attain.

2 Evidence on the Effectiveness of Popular Investment Strategies

It is important to note that popular prescriptions for investment success are standardized prescriptions for 'everyman', that generally do not take account of the details of the individual situation and, in particular, do not take account of the tastes of the individual as represented, for example, by the risk aversion of his von Neumann-Morgenstern utility function. In contrast, modern scientific investment rules[8] are predicated upon the precise specification of investor utility functions, even though it may be very difficult in practice to elicit such utility functions from the individual. Thus we should not expect the popular prescriptions to be 'optimal' for any given utility function, though we might expect them to be robust in the sense that they are reasonably close to optimal for a broad range of utility functions if the utility functions are 'reasonable'.

Perhaps the most fundamental decision faced by the investor is the appropriate allocation to risky securities. Early modern financial theory[9] makes this decision a complex function of the distribution of returns and investor tastes, while the simple 60-40 rule prescribes a simple 60% allocation to equities and 40% to cash. Brennan and Torous (1999) show that this simple prescription attains in practice a large fraction of the potential gains from investing in equities for a broad range of risk aversion. More generally, DeMiguel *et al.* (2005) show that when there are N asset classes, a simple $1/N$ asset allocation rule generally outperforms more sophisticated rules bases on financial theory.

A more sophisticated variant of the 60-40 rule is the prescription that the percentage of his wealth that an investor should allocate to equities is equal to 100 minus his age in years.[10] This age dependent allocation rule conflicts with early analytic models of dynamic investment planning such as that developed by Samuelson (1969) which prescribe that, at least for standard iso-elastic utility

[8] See for example Campbell and Viceira (2002)
[9] Standard mean-variance analyses of the Markowitz (1959)-Sharpe (1963) type.
[10] This rule is advocated for example by Malkiel (1996, p 418).

functions, the optimal equity allocation is age independent.[11] However, more sophisticated analytic models such as Bodie *et al.* (1992) and Cocco *et al.* (2004), which take account of the depreciating endowment of human capital, tend to validate the principles, if not the parameters, of the heuristic age dependent rule. For example, Cocco *et al.* (ibid.) show that for their base case parameters, the 100 minus age rule cuts the welfare loss of the simple Samuelson (ibid.) rule by around 58%.

In a paper that stimulated broad interest, Canner *et al.* (1997) questioned the more detailed asset allocation rules that have been proposed by popular investment advisors. They noted that, in contrast to implications of the standard Tobin Separation Theorem, the investment advisors recommended that more risk averse investors have proportionately larger allocations to bonds. However, the Tobin Separation Theorem is derived in a *static* setting and, as Brennan and Xia (2000) among others have shown, the advice of the popular investment advisors is easily rationalized in a dynamic setting when the interest rate is stochastic. In this case, the popular advice seems to have been based on a more sophisticated, albeit implicit, model than the model its academic critics were employing.

The popular advice to investors that they should cut their losses and let their profits run finds no support in the classical scientific theory of investment unless it is based on tax considerations.[12] Indeed, the random walk version of the efficient markets hypothesis and standard mean variance analysis would rather suggest the opposite: a *rebalancing* strategy in which winners are partially liquidated and losers further purchased in order to maintain portfolio efficiency. However, the evidence of price momentum that began to appear in the early 1990's, starting with Jegadeesh and Titman (1993), raised the possibility that perhaps the popular maxim was not without merit, even in the absence of tax considerations. More recent work by Odean (1998) has established that, not only is it more profitable to follow the strategy implied by the maxim, but that individual investors have a tendency to do the opposite - that is, to sell winners and to hold on to losers. Thus, once again, the folk-lore strategy, although contrary to the modern 'scientific' view, has been vindicated by more careful analysis of the actual behavior of prices in securities markets.

[11] Samuelson (1989) has been at pains to underline the error of an equity allocation that reduces with age which he attributed to a misunderstanding of the nature of risk.

[12] See Constantinides (1983) and Dammon *et al.* (2001).

In summary, there is evidence that popular heuristics or maxims for personal portfolio management have in some cases anticipated the findings of scientific studies of financial markets. One of the most popular heuristics is the Dollar Cost Averaging strategy: a Google search for this term yielded no less than 34,700 sites.

3 Dollar Cost Averaging

Dollar Cost Averaging (DCA) is a strategy for the purchase of equity securities that is widely recommended by professional investment advisors. The strategy calls for the periodic investment of a fixed amount of money in a stock or portfolio each period over a given time interval, instead of investing the whole amount in a lump sum at time zero. For example, an investor who wishes to invest $10,000 in stock XYZ may be advised to invest $1000 per month for ten months instead of investing the whole $10,000 at one time. The strategy may be applied on a stock by stock basis as the previous example suggests, or it may be applied to investment in stocks as an asset class; for example, an individual who receives an inheritance of $500,000 may decide to invest $100,000 per year in stocks for each of five years, keeping the balance of his money in low risk money market or bond accounts. The intuition behind Dollar Cost Averaging is that it allows the investor to purchase stock at an average price which is below the average of the prices prevailing on the purchase dates because the fixed periodic dollar investment purchases more shares when the price is low.

DCA is similar to other mechanical plans for investing such as portfolio rebalancing or the portfolio insurance type ofstrategies which Brennan and Solanki (1981) and Leland (1980) have shown to be optimal under certain circumstances. The major distinction between DCA and these strategies is that DCA is a non-contingent strategy, whereas the hallmark of portfolio insurance and asset allocation type strategies (including rebalancing to constant proportions) is that the investments are contingent on the underlying stock price.

3.1 The Popular View

While discussions of DCA are now generally omitted from standard investments texts, presumably because they are not thought worthy of serious discussion, a textbook discussion of DCA can be found, for example, in Francis (1980) where it is described (page 706) as a 'simple investment plan which helps uninformed investors with the timing of their investments.' The author continues (page 707) that 'one of the keys to earning high returns is to buy when the market is depressed. However,.., amateur investors often sell at market lows. For people who lack the time or ability to forecast market returns or who become upset during bear markets, the dollar averaging plan might help them buy when the market is down and stocks are cheap - *if* they stick to the plan.' It is interesting to note that Francis' account relates both to issues of self-control and to mean reversion in stock prices, although neither of these phenomena was recognized by academics in finance at the time the text was written. Similarly, Latane et al. (1975), page 517) describe DCA as a 'technique for taking advantage of stock market ups and downs.' Another textbook of the same era written by Cohen *et al.* (1977, page 48) also stresses the element of self-control: 'the important thing is to stick to your schedule..which, pyschologically, is usually hard to do.' Even Sharpe's (1978) classic text concludes that 'There is nothing inherently wrong with dollar averaging, but it has no magical properties. Dollar averaging certainly does not change uncertainty *per se* from a vice to a virtue.' (page 578)

Despite the fact that contemporary *academic* texts no longer contain any discussion of DCA, it is still a strategy that is widely advocated in more popular publications. For example, Malkiel's (1996) *Random Walk Down Wall Street*, which is a popular distillation of academic advice on investment tempered with some practical wisdom, has a section entitled *Dollar-Cost Averaging can Reduce the Risks of Investing in Stocks and Bonds.* In it, he claims (page 356) that 'This technique is controversial, but it does help you avoid the risk of putting all of your money in the stock or bond market at the wrong time.' Interestingly, he does not explain why the technique is controversial, but goes on to say that '(DCA) can reduce (but not avoid) the risks of equity investment by ensuring that the entire portfolio of stocks will not be purchased at temporarily inflated prices' and presents an historical table which illustrates the 'tremendous *potential* gains possible from consistently following

(DCA).' Malkiel also stresses the psychological difficulty of actually following through on a DCA strategy 'no matter how pessimistic you are.' It is clear that his rationale for the strategy is based on a model of 'market panics' and 'speculative explosions.'

3.2 The Academic View

In the pre-modern era there was significant interest among academics in 'formula plans' for investing which included DCA as a special case. For example, the analyses of Ketchum (1947) and Weston (1949) are predicated on the notion that stock prices are 'cyclical'. Ketchum analyses the returns to plans that alter the fund exposure to equity as a function of the level of the stock index relative to its trend value (fitted *ex-post*) and, not surprisingly, finds that formula timing plans are superior to 100% equity strategies, while Weston analyzes different formula plans in terms of the types of price fluctuations which are implicitly being assumed. Cottle and Whitman (1951) discuss historical simulations of formula plan returns, although they only present results for a constant bond-stock mix. Not surprisingly, interest in formula plans waned as the random walk replaced cyclicality as the standard model of stock price behavior.[13]

During the era in which the random walk theory was widely accepted as a good approximation of stock market behavior, the few academic articles on DCA that appeared were uniformly negative. Pye (1971), Neave and Wigginton (1976) and Constantinides (1979) criticize DCA on the grounds that it makes the investor's portfolio at any point in time dependent on its initial composition, so that two investors with identical tastes and wealth who follow DCA strategies will hold different portfolios at any point in time if the initial composition of their portfolios is different. But for an expected utility maximizing investor the optimal portfolio should depend only on current information. Brennan and Solanki (1981) prove that DCA is suboptimal for an investor maximizing the expected utility of terminal wealth when returns are *iid*. Rozeff (1994) demonstrates that, after adjusting appropriately for risk within a mean variance framework, a lump sum investment policy is always superior to a DCA policy if the stock market has a positive risk premium. However, his proof rests

[13]The random walk hypothesis emerged in the papers of Kendall (1953), Roberts (1959), and Alexander (1961) which are collected in Cootner's (1964) classic *The Random Character of Stock Market Prices*

on the assumption that successive market returns are independent. Dybvig (1988) presents a more general approach for calculating the inefficiency cost of policies similar to DCA for arbitrary utility functions, but again his analysis is predicated on the assumption of *iid* returns.

The random walk hypothesis is now generally rejected as an adequate description of stock price behavior.[14] Moreover, the psychological problems faced by an investor operating in highly uncertain environment that are stressed in most popular accounts of DCA are now receiving increased attention from academics.[15] However, these developments have yet to spark a renewed academic interest in formula plans, although Statman (1995) provides a behavioral interpetation of dollar cost averaging and argues (page 75) that "the non-sequential rules of dollar-cost averaging reduce responsibility and regret". In what follows we present empirical evidence on the performance of DCA strategies for an investor whose objective is to maximize the expected value of a standard von Neumann-Morgenstern utility function defined over wealth. Our emphasis is on the behavior of stock prices and their implications for DCA. We ignore all issues that are related to modern developments in behavioral finance, not because we think that such issues as self-control and confidence in the underlying model are not important, but because they are too broad for us to introduce here.

4 Data

Security prices and firm market values are from CRSP for the period from December 1925 to December 2003. The market portfolio is taken as either the CRSP Equal Weighted Market Portfolio or the CRSP Value Weighted Market Portfolio, and the riskless interest rate is the 30-day T-bill rate, also taken from CRSP. A security is considered to be available for investment in a given month if the security price is available at the end of the previous month. The total number of individual securities is 25,396. The maximum number of securities available in any month is 9,265; the minimum is 513, and the median is 2,143.

Stocks that are delisted without a delisting return being given by CRSP are assigned a return

[14] Summers (1986) was among the first to point out the weakness of the empirical foundations of the random walk; De Bondt and Thaler (1985), Poterba and Summers (1988) and Jegadeesh and Titman (1993) among others present evidence of 'mean reversion' in stock price behavior.

[15] For a survey of this literature see Hirshleifer (2001).

of -100% if the delisting is due to liquidation or for performance reasons, or if the issue stopped trading for unknown reasons;[16] if the delisting return is due to a merger or exchange offer without detailed information being available[17] a delisting return of zero is assigned. After the delisting, the proceeds (if any) are assumed to be invested in the risk free asset until the horizon and no further investments in the risky asset are made.

5 Simulation

In order to assess the relative merits of the DCA strategy, Monte Carlo simulations were run for purchases of the market portfolio and of individual securities. For each strategy s the expected utility was calculated for a power utility function of the form:

$$U(W_s) = \frac{{W_s}^{1-\gamma}}{1-\gamma} \tag{1}$$

for different values of γ, the coefficient of relative risk aversion. We analyze, in turn, DCA strategies for investing in the market portfolio, for purchasing a single stock when no stocks are held initially, and for purchasing a single stock when a portfolio of common stocks is already held.

5.1 Investing in the Market Portfolio

To evaluate the T month dollar cost averaging strategy of investing in the market portfolio, the proceeds of an immediate lump-sum purchase of the market portfolio, and the proceeds of the dollar cost averaging strategy, were calculated for investments of \$1, starting in $10,000$ randomly chosen months that were selected with equal probability between December 1925 and January 1998. For each simulation, the wealth relatives for a buy and hold strategy of investing 100% of wealth in the market portfolio for $T=$ 12, 24, 36,48, 60, and 72 months were calculated. The corresponding wealth relative for the T-month dollar cost averaging strategy was calculated by assuming that at the beginning of the first month a fraction $1/T$ of the wealth was invested in the market portfolio and the balance in the risk free asset. Then, at the end of each month $j = 1, \cdots, T-1$, a fraction

[16]CRSP delisting code (DLSTCD) of 400-599 or 160.
[17]DLSTCD of 200-399. Shumway (1997) reports average delisting returns of minus 30%.

$1/(T-j)$ of the remaining wealth in the riskless asset was invested in the market portfolio. If the riskless interest rate is equal to zero, this strategy implies that an additional fraction $1/T$ of the initial wealth is invested in the risky asset each period. Thus the fraction of wealth left in the riskless asset at the end of the first period is $(1-\frac{1}{T})(1-\frac{1}{T-1}) = \frac{T-2}{T}$, and so on. This corresponds exactly to what is meant by a dollar cost averaging strategy. With a positive interest rate, the fraction of the initial wealth that is invested in the market increases at the interest rate. Let W_{isT} denote the wealth at the end of T months in simulation i from investing \$1 under strategy s. The certainty equivalent wealth for strategy s, $CE_s(T)$ is defined as the amount of wealth to be received with certainty at time T which would make the investor as well off as following strategy s. It is defined by:

$$CE_s(T) = \left\{ \frac{1}{10,000} \left[\sum_{i=1}^{10,000} W_{isT}^{1-\gamma} \right] \right\}^{\frac{1}{1-\gamma}}. \tag{2}$$

Panel A of Table 1 reports the certainty equivalents for four strategies for investing in the *value-weighted* market portfolio for γ ranging from 2 to 7. The first is to buy and hold the market portfolio (BH): under this strategy the whole of the initial dollar of wealth is assumed to be invested in the market portfolio at time zero. The second is the dollar cost averaging strategy described above (DCA). The third (50:50) is the strategy of investing 50% of the initial wealth in the market portfolio at time zero and the balance in the riskless asset, and then *rebalancing* every month to maintain the portfolio proportions. The fourth (OPT) is to invest the *'optimal'* proportions in the market and the risk free asset and to *rebalance* to maintain the proportions. The optimal allocation to the market under the assumption of continuous rebalancing and *iid* returns, x^*, is given by:

$$x^* = \frac{(\bar{R}_M - R_f)}{\gamma {\sigma_M}^2} \tag{3}$$

where $\bar{R}_M - R_f$ is the mean return on the market portfolio in excess of the riskless interest rate, and σ_M is the volatility of the excess return. These parameters were estimated from the sample data and were 7.4% and 19.1% for the value weighted market portfolio, and 11.5% and 26.1% for the equal weighted market portfolio. Note that what we have called the optimal allocation will not be truly optimal in that the rebalancing is at discrete intervals and market returns are not *iid.*

In fact, the certainty equivalent under the 'optimal' strategy does exceed that under the other strategies in almost all cases. However, the optimal strategy is predicated on the assumption that the investor knows, not only his risk aversion coefficient, but also the exact distribution of the excess returns; it is therefore practically infeasible. The buy and hold strategy has the most variable certainty equivalent across risk aversion levels; for the 36 month strategy, it falls from 1.26 for a risk aversion of 2 to 0.48 for a risk aversion of 7. In contrast the DCA strategy is much more robust across risk aversion levels, falling from 1.22 for RRA=2 to 0.87 for RRA equal 7. However, the DCA strategy is uniformly dominated by the 50:50 strategy.[18] It is interesting to note that the expected wealth under DCA is almost identical to that under the 50:50 strategy: it follows that the 50:50 strategy has less risk than the DCA strategy. These results are consistent with what one would expect in a random walk market with no risk premium since then both the DCA and the 50:50 strategies would imply that *on average* 50% of wealth was invested in stocks. However, the 50:50 strategy achieves better time diversification since under this strategy exactly 50% of wealth is invested in stocks each month, whereas under DCA the expected proportion of wealth in stocks increases over time. Thus, there is no evidence that the claimed market timing advantage of DCA, that it allows the investor to 'avoid the risk of putting all of his money in the stock market at the wrong time,' is sufficient to offset the poor time-diversification that it causes.

Panel B reports corresponding results for the *equal weighted* market portfolio. The results are similar. Thus there is no evidence that dollar cost averaging represents a good approach to investing in the market portfolio.

5.2 Buying Individual Securities

Table 2 reports results that are similar to those reported in Table 1 except that, instead of investing in the market portfolio, the investor is assumed to invest in a *single* security. The table presents the certainty equivalents for two strategies: BH is the strategy of investing 100% of available funds in the chosen security at the end of the randomly chosen month. DCA is the strategy of buying the

[18]We also considered a 50:50 buy and hold strategy which slightly outperformed the monthly rebalancing strategy whose results are reported in Table 1.

same security by dollar cost averaging as explained above. Panel A reports the results when the security to be purchased is chosen from all the available securities with *equal probability*. Panel B reports the results when the security is chosen with a probability that is proportional to the market value of the security outstanding on the trade date. Returns on the two strategies are calculated in the same manner as for the market purchases just described, allowing for the delisting of a security. When the security is delisted, the proceeds that are calculated in the manner described in Section 4 are assumed to be invested in the risk free asset.

Three features are apparent from the figures in Table 2. First, most of the certainty equivalents are less than unity, implying that investment in a single security leads to an economic loss even without taking account of the time value of money represented by the interest rate. The reason for this of course is the very poor diversification, and correspondingly low level of portfolio efficiency, achieved with a single stock portfolio.[19] Thus it is generally uneconomic to purchase a single stock portfolio either outright or by a dollar cost averaging strategy. However, such a purchase may make sense for a young person who is beginning to build a portfolio of individual stocks.[20] Secondly, the certainty equivalents tend to be significantly higher when the stock is chosen with a value weighted probability. The reason for this is the lower level of total risk of large firms,[21] and the correspondingly higher level of portfolio efficiency they offer for a single stock portfolio. Thirdly, *in all cases* the certainty equivalent of the DCA strategy exceeds that of the corresponding BH strategy. This is in part due to the fact that the DCA strategy involves a lower average investment in the security and, as we have seen, on average investment in a single risky security portfolio is uneconomic. However, there appears to be more to it than this. The certainty equivalent almost always decreases with the horizon for the BH strategy; but for DCA the certainty equivalent increases with the horizon for low levels of risk aversion, and more so for value weighted security selection. For $\gamma = 2$ the certainty equivalent grows with the horizon at a rate marginally above the riskless interest rate which averaged 4% during our sample period.

[19] Brennan and Torous (1999) show that the certainty equivalent wealth from investing in a randomly selected single security portfolio for 10 years is $0.36.

[20] It is known that a personal stock portfolio may have tax advantages over a mutual fund.

[21] See Campbell *et al.* (2001).

5.3 A Portfolio Approach

The analysis of the previous section is limited by the fact that a single security portfolio is inherently inefficient. In this section therefore we consider an investor who is adding a new security to an existing portfolio. We explicitly consider adding only a single security to an existing portfolio, and compare the value of doing this by way of a (marginal) lump sum purchase on the decision date (BH), and doing it by periodic fractional purhcases under a DCA strategy. For many individual investors these are the realistic alternatives. Moreover, to the extent that our measure shows that DCA dominates the BH strategy on an individual security basis, it will also dominate for the purchase of (a marginal amount of) of a portfolio of securities. We assume that the existing portfolio is the market portfolio. While the assumption that the investor holds the value weighted portfolio is a natural one since it flows from the logic of the capital asset pricing model, the gain from adding a new security to a portfolio that is already well diversified is likely to be small. Therefore we shall also consider the case in which the investor holds the equally weighted market portfolio, which in the US context corresponds better to the case in which the investor's initial portfolio is not well diversified.

Let $\tilde{W}_T^e$ denote the wealth at the end of month T of an investor under his chosen or equilibrium investment strategy e, and consider the marginal value of \$1 invested in a strategy s whose payoff at time T is a random variable, $\tilde{y}_{sT}$. We denote this marginal value of strategy s at time T by $MV_s(T)$, where

$$MV_s(T) = E\left[U'(\tilde{W}_T^e)\tilde{y}_{sT}\right]. \tag{4}$$

Define the Marginal Value Ratio (MVR) of the DCA strategy relative to the BH strategy for a given investment horizon T by:

$$MVR(T) = \frac{MV_{DCA}(T)}{MV_{BH}(T)}. \tag{5}$$

If the investor who holds an existing portfolio with payoff $\tilde{W}_T^e$ is indifferent between the DCA and BH strategies then $MVR(T) = 1$. $MVR(T) > 1 (< 1)$ implies that the DCA strategy dominates (is dominated by) the BH strategy.

We calculate the empirical counterpart to expression (5) for a T month horizon by randomly selecting for each simulation i a starting month and a security to invest in. The compound value of \$1 invested in the market portfolio for the next T months, W^e_{iT}, is calculated, along with the payoff from investing in the chosen security under DCA and BH, which we denote by $y_{DCA,i,T}$ and $y_{BH,i,T}$. Then the realized marginal value for strategy s (s = DCA,BH) is calculated by $RMV_{is} = U'(\tilde{W}_{ieT})\tilde{y}_{isT}$, and the estimated marginal value ratio is calculated as:

$$\widehat{MVR}(T) = \frac{\sum_{i=1}^{10,000} RMV_{i,DCA}}{\sum_{i=1}^{10,000} RMV_{i,BH}}. \tag{6}$$

Table 3 reports values of $\widehat{MVR}(T)$ for different values of the risk aversion coefficient. In Panel A the securities are assumed to be selected with equal probability from those outstanding in the given month. In Panel B the probability of selection is proportional to the firm market value. The figures in bold are based on the assumption that the investor holds the value weighted market portfolio and the figures below those on the assumption that the investor holds the equal weighted market portfolio.

All of the figures in the table up to the 48 month horizon are greater than or equal to unity, and most of them are considerably in excess of unity. For the 36 month horizon, the marginal value for an investor who holds the market portfolio, of purchasing a random security by DCA exceeds that of an immediate lump sum purchase by from 5% to 134%, depending on the assumption about risk aversion, the weighting of the market portfolio held and whether the security is chosen with equal or value weighted probability. We conclude that any investor who holds the (equal or value-weighted) market portfolio would be better off following a dollar cost averaging strategy in purchasing a new security than in purchasing the security outright. The advantage of DCA is greater when the security to be purchased is selected with equal probability. This is consistent with smaller stocks having returns that are more 'anomalous'. Moreover, when the securities for purchase are selected with equal probability the advantage of DCA is greatest for an investor who holds the equally weighted market portfolio, and when the securities for purchase are selected with value weighted probability the advantage of DCA is greatest for an investor who holds the value weighted market portfolio.

The relative advantage of the DCA strategy is maximized when it is executed over a 36-48 month horizon.

We found in analyzing purchase strategies for the market portfolio that, while DCA dominated BH, both strategies were dominated by a strategy of maintaining a constant 50% of wealth in the market portfolio. Therefore Table 4 repeats the analysis of Table 3 but with W^e_{iT} being determined by a 50:50 mix of the riskless asset and the market portfolio rebalanced monthly. The effect of reducing the riskiness of the investor's pre-existing portfolio is to reduce the marginal value ratios, which suggests that part of the (relative) marginal value of the dollar cost averaging strategy comes from its marginal effect on reducing the risk of the pre-existing portfolio when that exceeds the investor's optimal level. The marginal value ratio for the 36 month strategy exceeds unity for all values of relative risk aversion greater than or equal to 4. We suspect that few investors in practice hold an optimally levered position in stocks, and the evidence in Tables 3 and 4 suggests that dollar cost averaging remains good advice for a broad range of investors and, even when it is dominated by the BH strategy, the margin is relatively small except for relative risk aversion of 2 where it can be as high as 10% for the $T = 36$ months strategy when the portfolio leverage is 50%.

Finally, in order to determine whether the whole advantage of DCA in the purchase of individual securities arises from its risk reducing effect on an initially excessively levered portfolio, the analysis was repeated assuming that the investor's initial portfolio is optimally levered. That is, the fraction of the portfolio allocated to equities is assumed to be given by equation (3), and the portfolio is rebalanced monthly to maintain the optimal weighting. This is an extremely conservative assumption because we are assuming that the investor could have formed an optimal portfolio based on the sample moments of portfolio returns over the whole period even though these were not available until the end of the sample period. The results are reported in Table 5. The results for the 36 month DCA strategy may be summarized as follows. When an investor holds an optimal mix of the *value weighted* market portfolio and the riskless asset, a 36 month DCA strategy for purchasing a new security yields a marginal advantage of $3 - 6\%$ over a traditional BH strategy. In contrast to the results reported for non-optimally levered initial portfolios, the advantage of DCA appears to be marginally greater when the securities for purchases are selected with value weighted probability;

this suggests that the effect is greatest for *large* firms. Curiously, the effect largely disappears, and is even reversed for long horizons, when the investor is assumed to hold an optimally levered *equal weighted* market portfolio.

The continuing advantage of the DCA strategy even when the initial portfolio is optimally levered shows that its advantage does not arise from an inappropriately levered initial portfolio. The strategy appears to have a fundamental benefit arising from unknown properties the autocorrelation and lagged cross-correlation of returns.[22] A general property of our results is that the advantage of the DCA strategy for adding a new security to an existing portfolio is maximized when the strategy is executed over a horizon of 36-48 months. In the following section we shall show that it is possible to account for some of the characteristics of our DCA results by a model of stock price behavior which allows the market price of a security to depart from its fundamental value; however, we are not able to explain the maimum at 36-48 months.

6 A Model of Stock Price Behavior

[23]

Following the general rejection of the random walk hypothesis, there is no longer a canonical model of individual stock price behavior which can be used to explain the success of the DCA strategy for adding a new stock to a portfolio. Moreover, no single model is likely to appropriate for securities of different size firms with different risk and information charactersitics. Nevertheless, to gain some insight we shall extend the model which Poterba and Summers (1988) use to account for mean reversion in stock prices, so that it allows also for some short run positive serial correlation in returns. Thus, following Poterba and Summers, we write the stock price at time t, P_t, as the product of the fundamental value, P_t^*, and a stationary process, U_t:

$$P_t = P_t^* U_t \tag{7}$$

[22] Properties of the lagged cross-correlations of returns have been studied by Lo and MacKinlay (1990), Brennan *et al.*(1993), Chordia and Swaminathan (2002).

[23] We thank a referee for suggesting this analysis.

where $u_t \equiv lnU_t$ follows the process:

$$u_t = \rho u_{t-1} + \sigma_u \left(z_{u,t} + \eta_1 z_{u,t-1} + \eta_2 z_{u,t-2}\right) \tag{8}$$

and the fundamental value, P_t^*, is assumed to satisfy the Capital Asset Pricing Model, so that the fundamental return, $\tilde{R}_t^* \equiv (\tilde{P}_{t+1}^* - P_t^*)/P_t^*$, satisfies:

$$\tilde{R}_t^* = R_{F,t} + \beta(\tilde{R}_{M,t} - R_{F,t}) + \epsilon_t \tag{9}$$

where $z_{u,t}$ and ϵ_t are standard normal variables, $E[z_{u,t}] = E[\epsilon_t] = E[z_{u,t}\epsilon_t] = 0$, and $R_{M,t}, R_{F,t}$ are the returns on the market portfolio and risk free asset at time t. The ARMA(1,2) model (8) allows for positive correlation at short horizons. When $\eta_1 = \eta_2 = 0$ the model reduces to the AR(1) model used by Poterba and Summers. Market and stock returns are generated monthly using equations (7),(8), and(9), using the parameter values given in Panel A Table 6. The parameters $R_F, E[R_M], \sigma_M^2$, were set equal to the sample mean of risk free rate and the mean and variance of the CRSP value weighted index over the period 1926-2003. The security specific parameters were chosen by trial and error to generate a pattern of positive serial correlation for short horizons followed by negative serial correlation, and to correspond roughly to the pattern of marginal value ratios seen in Table 6. For the base case the security specific parameter values imply that the standard deviation of the (log of the) market price to fundamentals ratio, $\tilde{U}$, has a standard deviation of 38.4%.[24] The analysis of Section 5.3 whose results were reported in Table 5 was repeated, this time using 25,000 simulations for the market return and security specific variates, $z_{u,t}, \epsilon_t$. First it was assumed that the investor held an initial portfolio that was the optimal combination of the market portfolio and the risk free asset, given his degree of risk aversion. Then, to simulate an initial portfolio that was not fully diversified, it was assumed that his initial portfolio was the optimal combination of the risk free asset and a portfolio whose return was equal to the simulated return on the market portfolio plus a normally distributed return with mean zero and (annualized) standard deviation, σ_z equal to 5%. The marginal value ratios for various scenarios are reported in Panel B of Table 7. As before,

[24] Black (1984) suggests that an efficient market is one in which 'price is within a factor of 2 of value'.

a marginal value ratio in excess of unity implies the superiority of the DCA strategy relative to BH. The figures in boldface relate to an investor who initially holds the optimal mix of the risk free asset and the market portfolio; the figures below them to an investor who holds the less well diversified portfolio. The results were found to be largely insensitive to the assumed value of relative risk aversion, γ, and the table reports results only for $\gamma = 2$. Model 1 uses the base parameter values and the other models use the base values except for the parameter noted.

Panel B of the table shows that the autocorrelations of returns are positive for the first lags (0.06) and then turn negative, peaking at -0.07 at lag 3. Campbell, Lo, and MacKinlay (1997) report autocorrelations of 0.043 (0.171), -0.053 (-0.034),-0.013 (-0.033),-0.004 (-0.016) for lags 1-4 for monthly returns on the CRSP value (equal) weighted portfolio for the period 1962-94. It is to be expected that autocorrelations for portfolios will be more positive than for the individual securities because of thin trading and liquidity effects.

Note first that, when $\sigma_u = 0$ so that the price is always equal to fundamental value and the return therefore satisfies the CAPM, the investor is essentially indifferent between the DCA and BH. The reason for this is that an investor holds an optimally levered market portfolio is indifferent at the margin between an investment in the riskfree asset and a security that satisfies the CAPM. When the investor holds the optimally levered imperfectly diversified portfolio the DCA strategy is slightly inferior to BH but the difference is not economically significant except for very long execution periods.

For the base case parameters, the advantage of DCA is of the order of 6% to 7% when the execuion period is 4-5 years, which compares with the 7-9% reported in Table 5 for $\gamma = 2$. However, while we were able to reproduce the magnitude of the gains found with actual returns, we were not able to reproduce the attenuation in the gains at long horizons found with the actual returns. This may be due to the restrictive nature of the pricing process that we have assumed which induces only only short run dependence in returns. It may also be due to the restriction in the simulations to only a single process for all securities; as noted above, the process is likely to different for securities that differ in firm characteristics. For example, Brennan *et al.* (1993) have found that the prices of small stocks tend to adjust more slowly to economy wide information, and Chordia and Swaminathan (2000) have found that the returns on portfolios of high trading volume portfolios lead those of low volume portfolios after adjusting for firm size.

As the parameters of the mispricing process were varied from the base case we found that an increase (decrease) in the lag 1 autocorrelation correlation, induced for example by increasing (decreasing) ρ or η_1, tends to increase (decrease) the advantage of DCA, and increasing the lag 2 autocorrelation by increasing η_2 also tends to increase the value of DCA. Of course, these generalizations apply only for local variation in the parameters around the base case.

In summary, we have been able to reproduce the gross features of the DCA results in a model of security mispricing that based on the Poterba Summers model of transitory security mispricing.

7 Summary

In this paper we have analyzed the empirical properties of dollar cost averaging portfolio and security purchase strategies. While such strategies are widely recommended by popular investment advisors, they find no mention in the standard academic texts that are currently used to train MBA and undergraduate students in finance. The only recent references that we have found to dollar cost averaging in the scholarly literature emphasize the suboptimal nature of such strategies.

We have evaluated the usefulness of dollar cost averaging strategies by simulating a large number of portfolio and security purchases over the period January 1926 to December 2003 and comparing the certainty equivalent returns for an investor with power utility of two types of strategy. The first strategy, which we call buy and hold (BH), purchases the security or portfolio in a randomly selected month and holds it for a fixed T month period. The second strategy is the dollar cost averaging strategy (DCA), in which equal amounts (in present value) are invested in the security in each of T months.

When DCA is applied to purchases of the market portfolio, the results are heavily dependent on the assumed risk aversion of the investor. DCA is superior to BH for more risk averse investors, but only because such investors should not be investing the whole of their wealth in the market portfolio. DCA is dominated by a BH strategy in which the investor invests a fraction of his wealth in the market portfolio that is *optimal* given his risk aversion and the sample moments of the return distribution. Of course, unlike the DCA strategy, the optimal strategy cannot be determined *ex-ante* since it relies on estimates of the moments of the return distribution that are estimated from the whole data sample. However, for all levels of risk aversion the DCA strategy is also dominated by a strategy in which 50% of wealth is invested in the market portfolio and the portfolio is rebalanced monthly to maintain the proportions. This result is broadly consistent with what we would expect in a random walk market, for it is well known that an investor with power utility in such a market should optimally maintain a constant fraction of his wealth in the market portfolio. The 50:50 strategy roughly approximates this.[25] The DCA strategy has a strongly time varying allocation to the risky asset that fails to achieve efficient 'time-diversification' so that the investor's final wealth is much more heavily dependent on the return in the last month of the period than it is on the return in the first month.

When DCA is applied to the purchase of a single security it easily dominates a BH strategy, but this is only because of the inefficiency of a single security portfolio. The advantage of the DCA strategy lies in the fact that it is less heavily invested in this inefficient portfolio.

A more reasonable interpretation of the DCA prescription is that it should be applied to the purchase of securities that are being added to portfolios that are already well diversified. We therefore compute the marginal utility of the payoffs to simulated DCA and BH strategies for the investment of $1 in a security and then calculate the ratio of the expected marginal utilities of the two strategies. We find that the DCA strategies always dominate for an investor whose initial portfolio is the market portfolio, either value weighted or equal weighted, for DCA execution periods of up to 48 months: the advantage of a 3 year DCA strategy ranges from 5% to 134%, depending on the risk aversion of the investor. In this sense DCA is a good strategy for the average investor. When the initial portfolio is changed to a 50:50 mix of the market and the riskless asset, the advantage of DCA is mitigated; however, the qualitative advantage of DCA is maintained, except for investors with very low risk

[25] Brennan and Torous (1999) show that modest departures from the optimal portfolio mix have only second order effects on expected utility.

aversion ($\gamma = 2$ or 3). In order to determine whether the relative advantage of DCA arises from its 'delevering' effect on an initial portfolio that is over-levered relative to the investor's assumed risk aversion, the analysis was repeated assuming that the investor's initial portfolio was optimally levered, where the optimal leverage is determined using *full sample* data on portfolio returns. Despite this 'super-efficiency' of the investor's initial portfolio, DCA retains its advantage for all levels of risk aversion considered when the underlying market portfolio is value-weighted; its advantage over the standard BH strategy is of the order of $3 - 6\%$ at the 36 month horizon where the advantage of the DCA strategy is greatest. An interesting implication of the superiority of DCA for purchasing a marginal stock is its inferiority for selling a stock. Liquidity considerations apart, individual stocks should be sold in a block rather than liquidating a fixed number of dollars of stock each period.

We have shown using Monte Carlo simulation that the advantages of the DCA strategy for purchasing a security to add to an existing portfolio could be attributable to transient security mispricing of the kind proposed by Poterba and Summers (1988).

In summary, dollar cost averaging is a heuristic that has been almost entirely overlooked by academics since the development of the random walk hypothesis, and which is suboptimal under standard assumptions about capital markets. Despite this, the strategy appears to be useful in real world capital markets. We have not attempted to consider the role of transactions costs or market impact. Nor have we attempted to determine the characteristics of security price behavior that account for the relative success of the strategy. These are topics for more formal investigations. As mentioned above, our analysis considers the benefits of DCA only from the viewpoint of a 'rational' investor with a well-defined von Neumann-Morgenstern utility function. An important further line of enquiry is to consider this and similar strategies from the viewpoint of an investor who is subject to the behavioral biases that are currently receiving increased attention from academics.

8 References

Alexander, S.S., 1961, Price Movements in Speculative Markets: Trends or Random Walks, *Industrial Management Review*, 2, 7-26.

Black, F., 1984, Noise, *Journal of Finance*, 41 , 529-543.

Blume, L., and D. Easley, 1992, Evolution and Market Behavior, *Journal of Economic Theory*, 58, 9-40.

Bodie, Z., R.C. Merton, and W. Samuelson, 1992, Labor Supply Fleibility and Portfolio Choice in a Life-Cycle Model, *Journal of Economic Dynamics and Control*, 16, 427-449.

Brennan, M.J., and R. Solanki, 1981, Optimal Portfolio Insurance, *Journal of Financial and Quantitative Analysis*, 3, 279-300.

Brennan, M.J., 2004, How Did it Happen?, *Economic Notes*, 3, 3-33.

Brennan, M.J., N. Jegadeesh, and B. Swaminathan, 1993, Investment Analysis and the Adjustment of Stock Prices to Common Information, *Review of Financial Studies*, 6, 799-824.

Brennan, M.J., and W.N.Torous, 1999, Individual Decision Making and Investor Welfare, *Economic Notes*, 28, 119-143.

Brennan, M.M., and Y. Xia, 2000, Stochastic Interest Rates and the Bond- Stock Mix,*European Finance Review*, 4, 197-210.

Campbell, J.Y., A. Lo, and A.C. MacKinlay, 1997, *The Econometrics of Financial Markets*, Princeton University Press, Princeton NJ.

Campbell, J.Y., and L. Viceira, *Strategic Asset Allocation*, Clarendon Press, Oxford, 2002.

Campbell, J.Y., M. Lettau, B. Malkiel, and Y. Xu, 2001, Have Individual Stocks become more volatile?, *Journal of Finance*, 56, 1-43.

Canner, N., N.G.Mankiw, and D.N.Weil, 1997, An Asset Allocation Puzzle, *American Economic Review*, 87, 181-191.

Chordia, T., and B. Swaminathan, 2000, Trading Volume and Cross-autocorrelations in Stock Returns,*Journal of Finance*, 55, 913-936 .

Cocco, J.F., F.J. Gomes, and P.J. Maenhout, 2003, Consumption and Portfolio Choice over the Life-Cycle, *Review of Financial Studies*, forthcoming.

Cohen, Zinbarg, E.D., and A. Zeikel, 1977, J.B., *Investment Analysis and Portfolio Management*, 3rd. Edition, Irwin, Homewood.

Constantinides, G.M., 1979, A Note on the Suboptimality of Dollar-Cost Averaging as an Investment Policy, *Journal of Financial and Quantitative Analysis*, 14 , 443-49.

Constantinides, G.M., 1983, Capital Market Equilibrium with Personal Taxes, *Econometrica*, 51, 611-636.

Cootner, P.H., 1964, *The Random Character of Stock Market Prices*, M.I.T. Press, Cambridge.

Cottle, C.S., and W.T. Whitman, 1951, Testing Formula Plans, *Journal of Finance*, 6, 220-228.

Dammon, R., C. Spatt, and H. Zhang, 2001, Optimal Consumption and Investment with Capital Gains Tax, *Review of Financial Studies*, 14, 583-616.

De Bondt, W.F.M., and R. Thaler, 1985, Does the Stock Market Overreact?, *Journal of Finance*, 40, 793-805.

DeMiguel, V., L. Garlappi, and R. Uppal, 2005, How Inefficient is the Simple $1/N$ Asset-Allocation Strategy?, unpublished manuscript, London Business School.

Dybvig, P.H., 1988, Inefficient Dynamic Portfolio Strategies or How to Throw Away a Million Dollars in the Stock Market, *Review of Financial Studies*, 1, 67-88.

Francis, J.C., 1980, *Investments*, 3rd. Edition, McGraw-Hill, New York.

Hens, T., R. Amir, I. Evstigneev and Klaus Schenk-Hopp, Market Selection of Financial Trading Strategies: Global Stability", joint with Working Paper No. 83, Institute for Empirical Research in Economics, University of Zurich forthcoming in *Journal of Mathematical Economics*, 2003.

Hirshleifer, D., 2001, Investor Psychology and Asset Pricing, *Journal of Finance*, 1533-1598.

Jegadeesh, N., and S. Titman, 1993, Returns to Buying Winners and Selling Losers, *Journal of Finance*, 48, 65-91.

Kendall, M.G., 1953, The Analysis of Economic Time Series –Part I: Prices, *Journal of Royal Statistical Association*, 96, 11-25.

Ketchum, M.D., 1947, Investment Management through Formula Timing Plans, *Journal of Business*, 20, 157-158.

Kurz, M.,1994a, On the Structure and Diversity of Rational Beliefs, *Economic Theory* 4, 877-900.

Kurz, M.,1994b, On Rational Belief Equilibria, *Economic Theory* 4, 859-876.

Latane, H.A., C.P. Jones, and D.L. Tuttle, 1975, *Security Analysis and Portfolio Management*, 2nd Edition, Ronald Press, New York.

Leland, H.E., 1980, Who Should Buy Portfolio Insurance?, *Journal of Finance*, 35, 581-594.

Lo, A., and C.A. MacKinlay, 1988, Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test, *Review of Financial Studies*, 1, 41-66.

Lo, A., and C.A. MacKinlay, 1990, When are Contrarian Profits Due to Stock Market Over-Reaction?, *Review of Financial Studies*, 3, 175-206.

Lo, A.,2002, Bubble, Rubble, Finance In Trouble?,*Journal of Psychology and Financial Markets*, 3, 76-86.

Malkiel, B.G., 1996, *A Random Walk down Wall Street: The Best Investment Advice for the New Century*, W.W. Norton Co., New York.

Markowitz, H., 1959, *Portfolio Selection: Efficient Diversification of Investments*, Wiley, New York.

Neave, E.H., and J.C. Wigginton, 1976, Dollar Averaging and Global Risk Aversion, School of Business, Queens University, WP76-7.

Odean, T., 1998, Are Investors Reluctant to Realize their Losses, *Journal of Finance*, 53, 1775-1798.

Poterba, J., and L.H. Summers, 1988, Mean Reversion in Stock Prices: Evidence and Implications, *Journal of Financial Economics*, 22, 27-60.

Pye, G., 1971, Minimax Policies for Selling an Asset and Dollar Averaging, *Management Science*, 17, 379-393.

Roberts, H.V., 1959, Stock Market "Patterns" and Financial Analysis: Methodological Suggestions, *Journal of Finance*, 14, 1-10.

Routledge, B.R., 1999, Adaptive Learning in Financial Markets, *Review of Financial Studies* 12, 1165-1202.

Routledge, B. R., 2001, Genetic Algorithm Learning to Choose and Use Information *Macroeconomic Dynamics* 5, 2001, 303-325.

Rozeff, M., 1994, Lump-Sum Investing versus Dollar Cost Averaging, *Journal of Portfolio Management*, 20, 45-50.

Samuelson, P.A., 1969, Lifetime Portfolio Selection By Dynamic Stochastic Programming, *The Review of Economics and Statistics*, 239-246.

Samuelson, P.A., 1989, The judgement of economic science on rational portfolio management: indexing, timing, and , long-horizon effects, *Journal of Portfolio Management*, 25, 4-12.

Shumway, T., 1997, The Delisting Bias in CRSP, *Journal of Finance*, 53, 327-340.

Solomon, E., 1948, Are Formula Plans what they seem to be? *Journal of Business*, 21, 92-97.

Sharpe, W.F., 1963, a Simplified Model for Portfolio Analysis, *Management Science*, 9, 277-293.

Sharpe, W.F., 1978, *Investments*, Prentice-Hall, Englewood Cliffs.

Statman, M., 1995, A Behavioral Framework for Dollar-Cost Averaging, *Journal of Portfolio Management*, 22, 70-78.

Summers, L.H., 1986, Does the Stock Market Rationally Reflect Fundamental Values?, *Journal of Finance*, 41,591-601.

Titman, S., and N. Jegadeesh, 1993, Returns to Buying Winners and Selling Losers: Implications for Market Efficiency,*Journal of Finance*, 48, xxxx

Weston, J.F. 1949, Some Theoretical Aspects of Formula Timing Plans, *Journal of Business*, 22, 249-270.

Table 1

Certainty Equivalent (CE) Payoffs per Dollar Invested for *Buy and Hold* (BH), *Dollar Cost Averaging* (DCA), 50% cash:50% stock (50:50), and *optimal* (OPT) strategies applied to the *Value Weighted* Market Portfolio (Panel A) and *Equal-Weighted* Market Portfolio (Panel B) purchases starting in randomly chosen months between December 1925 and January 1998 and continued for up to 72 months.

The 50:50 strategy invests half the wealth in the riskless asset and half in the market portfolio and rebalances monthly. The optimal strategy invests a fraction of wealth $x^* \equiv (\bar{R}_M - R_f)/(\gamma * \sigma_M^2)$ in the market portfolio and the balance in the risk free asset, where $\bar{R}_M - R_f$ is the mean excess return on the market portfolio, σ_M^2 is the variance of the return on the market portfolio. γ is the coefficient of relative risk aversion.

Panel A: Value Weighted Market Portfolio

		Horizon (months)					
Relative Risk Aversion	Strategy	12	24	36	48	60	72
2	BH	1.08	1.17	1.26	1.37	1.49	1.64
	DCA	**1.07**	**1.14**	**1.22**	**1.30**	**1.38**	**1.48**
	50:50	1.07	1.15	1.23	1.32	1.41	1.51
	OPT	1.08	1.16	1.25	1.36	1.49	1.64
3	BH	1.05	1.07	1.09	1.19	1.30	1.44
	DCA	**1.06**	**1.12**	**1.18**	**1.25**	**1.31**	**1.40**
	50:50	1.07	1.13	1.20	1.28	1.37	1.47
	OPT	1.07	1.13	1.19	1.28	1.38	1.50
4	BH	1.01	0.93	0.87	0.97	1.11	1.24
	DCA	**1.05**	**1.09**	**1.12**	**1.17**	**1.22**	**1.29**
	50:50	1.06	1.11	1.17	1.25	1.33	1.42
	OPT	1.06	1.11	1.17	1.24	1.33	1.43
5	BH	0.96	0.78	0.69	0.79	0.95	1.07
	DCA	**1.04**	**1.04**	**1.04**	**1.08**	**1.11**	**1.15**
	50:50	1.05	1.09	1.13	1.20	1.29	1.38
	OPT	1.05	1.10	1.15	1.22	1.29	1.38
6	BH	0.90	0.65	0.56	0.67	0.84	0.95
	DCA	**1.02**	**0.99**	**0.95**	**0.97**	**0.99**	**1.01**
	50:50	1.04	1.06	1.08	1.16	1.24	1.33
	OPT	1.05	1.09	1.14	1.20	1.27	1.34
7	BH	0.83	0.56	0.48	0.58	0.75	0.86
	DCA	**1.00**	**0.93**	**0.87**	**0.87**	**0.88**	**0.89**
	50:50	1.03	1.02	1.02	1.10	1.20	1.28
	OPT	1.05	1.09	1.13	1.19	1.25	1.31
Expected Wealth per Dollar Invested	BH	1.13	1.27	1.43	1.58	1.76	1.95
	DCA	**1.09**	**1.18**	**1.28**	**1.37**	**1.48**	**1.60**
	50:50	1.08	1.18	1.28	1.38	1.49	1.62

Panel B: Equal Weighted Market Portfolio

		Horizon (months)					
Relative Risk Aversion	Strategy	12	24	36	48	60	72
2	BH	1.09	1.19	1.30	1.44	1.63	1.86
	DCA	**1.08**	**1.17**	**1.26**	**1.36**	**1.48**	**1.63**
	50:50	1.08	1.18	1.28	1.39	1.52	1.67
	OPT	1.09	1.19	1.31	1.45	1.63	1.85
3	BH	1.04	1.04	1.05	1.18	1.37	1.55
	DCA	**1.07**	**1.13**	**1.20**	**1.28**	**1.37**	**1.50**
	50:50	1.07	1.15	1.24	1.34	1.47	1.61
	OPT	1.07	1.15	1.23	1.35	1.48	1.64
4	BH	0.99	0.87	0.78	0.89	1.12	1.25
	DCA	**1.05**	**1.08**	**1.12**	**1.17**	**1.23**	**1.32**
	50:50	1.06	1.12	1.19	1.29	1.41	1.55
	OPT	1.06	1.13	1.20	1.30	1.41	1.54
5	BH	0.93	0.71	0.58	0.69	0.92	1.03
	DCA	**1.03**	**1.03**	**1.02**	**1.05**	**1.07**	**1.11**
	50:50	1.05	1.08	1.12	1.23	1.36	1.48
	OPT	1.06	1.11	1.18	1.26	1.36	1.47
6	BH	0.86	0.59	0.47	0.56	0.77	0.87
	DCA	**1.01**	**0.97**	**0.92**	**0.92**	**0.92**	**0.94**
	50:50	1.04	1.04	1.05	1.16	1.30	1.42
	OPT	1.05	1.10	1.16	1.24	1.32	1.42
7	BH	0.80	0.51	0.40	0.48	0.67	0.76
	DCA	**0.98**	**0.91**	**0.84**	**0.82**	**0.81**	**0.81**
	50:50	1.02	1.00	0.98	1.09	1.24	1.35
	OPT	1.05	1.10	1.15	1.22	1.30	1.38
Expected Wealth per Dollar Invested	BH	1.18	1.39	1.60	1.87	2.14	2.42
	DCA	**1.12**	**1.24**	**1.36**	**1.50**	**1.65**	**1.83**
	50:50	1.11	1.22	1.34	1.48	1.62	1.78

Table 2

Certainty Equivalent (CE) Payoffs for *Buy and Hold* (BH) and *Dollar Cost Averaging* (DCA) strategies applied to randomly chosen securities and months of purchase selected with equal probability between December 1925 and January 1998. Under DCA the purchases are continued for up to 72 months, and under BH the securities purhcased are assumed to be held during the same period. Securities are chosen either with *Equal Probability* (Panel A) or *Value-Weighted Probability* (Panel B).

Panel A: Equal Probability Security Selection

		Horizon (months)					
Relative		12	24	36	48	60	72
Risk Aversion	Strategy						
2	BH	0.90	0.56	0.43	0.38	0.36	0.35
	DCA	**1.02**	**1.04**	**1.09**	**1.13**	**1.15**	**1.20**
3	BH	0.66	0.04	0.03	0.02	0.03	0.03
	DCA	**0.94**	**0.89**	**0.91**	**0.89**	**0.72**	**0.80**
4	BH	0.38	0.01	0.01	0.01	0.01	0.01
	DCA	**0.79**	**0.60**	**0.67**	**0.57**	**0.31**	**0.37**
5	BH	0.24	0.01	0.00	0.00	0.00	0.00
	DCA	**0.57**	**0.36**	**0.46**	**0.35**	**0.17**	**0.20**
6	BH	0.17	0.00	0.00	0.00	0.00	0.00
	DCA	**0.41**	**0.24**	**0.33**	**0.24**	**0.12**	**0.13**
7	BH	0.14	0.00	0.00	0.00	0.00	0.00
	DCA	**0.32**	**0.18**	**0.26**	**0.19**	**0.09**	**0.10**

Panel B: Value Weighted Security Selection

		Horizon (months)					
Relative		12	24	36	48	60	72
Risk Aversion	Strategy						
2	BH	1.01	0.96	0.88	0.89	0.88	0.96
	DCA	**1.04**	**1.09**	**1.15**	**1.21**	**1.27**	**1.36**
3	BH	0.91	0.46	0.09	0.10	0.10	0.10
	DCA	**1.02**	**1.02**	**1.06**	**1.11**	**1.12**	**1.20**
4	BH	0.74	0.17	0.02	0.02	0.02	0.02
	DCA	**0.99**	**0.92**	**0.92**	**0.98**	**0.91**	**0.95**
5	BH	0.54	0.09	0.01	0.01	0.01	0.01
	DCA	**0.94**	**0.76**	**0.75**	**0.82**	**0.68**	**0.69**
6	BH	0.41	0.06	0.01	0.01	0.01	0.01
	DCA	**0.87**	**0.61**	**0.61**	**0.69**	**0.52**	**0.51**
7	BH	0.33	0.05	0.00	0.00	0.00	0.00
	DCA	**0.78**	**0.50**	**0.50**	**0.59**	**0.41**	**0.40**

Table 3

Marginal Value Ratios between *Dollar Cost Averaging* and *Buy and Hold* purchase strategy payoffs for an investor who holds the market portfolio and commences purchases of randomly chosen securities in randomly chosen initial months between December 1925 and January 1998 and continues the purchases (DCA) or holds the securities for up to 72 months.

Securities are chosen either with *Equal Probability* (Panel A) or *Value-Weighted Probability* (Panel B). Figures in **bold** font relate to the **value-weighted** market portfolio, while figures in standard font relate to an **equally weighted** market portfolio. The Marginal Value Ratio is the ratio of the expected marginal utilities per dollar invested in the Dollar Cost Averaging and Buy and Hold strategies.

Panel A: Equal Probability Security Selection

	Horizon (months)					
Relative Risk Aversion	12	24	36	48	60	72
2	**1.00**	**1.00**	**1.05**	**1.05**	**1.05**	**1.02**
	1.02	1.05	1.10	1.08	1.03	1.00
3	**1.04**	**1.16**	**1.38**	**1.42**	**1.36**	**1.28**
	1.07	1.23	1.50	1.44	1.18	1.14
4	**1.11**	**1.47**	**1.90**	**1.96**	**1.71**	**1.59**
	1.14	1.55	1.99	1.90	1.24	1.21
5	**1.20**	**1.87**	**2.22**	**2.36**	**1.93**	**1.84**
	1.23	1.90	2.22	2.12	1.20	1.21
6	**1.34**	**2.14**	**2.32**	**2.54**	**1.98**	**1.99**
	1.33	2.13	2.29	2.16	1.13	1.18
7	**1.49**	**2.27**	**2.34**	**2.60**	**1.95**	**2.08**
	1.42	2.26	2.31	2.13	1.07	1.16

Panel B: Value Weighted Security Selection

	Horizon (months)					
Relative Risk Aversion	12	24	36	48	60	72
2	**1.01**	**1.03**	**1.06**	**1.06**	**1.04**	**1.02**
	1.01	1.04	1.05	1.02	0.96	0.93
3	**1.04**	**1.16**	**1.31**	**1.32**	**1.25**	**1.21**
	1.05	1.18	1.31	1.23	1.03	0.97
4	**1.09**	**1.38**	**1.66**	**1.69**	**1.48**	**1.40**
	1.10	1.39	1.61	1.48	1.08	0.94
5	**1.17**	**1.59**	**1.87**	**1.96**	**1.62**	**1.54**
	1.17	1.59	1.75	1.59	1.10	0.87
6	**1.28**	**1.70**	**1.95**	**2.08**	**1.68**	**1.61**
	1.24	1.70	1.80	1.59	1.10	0.81
7	**1.39**	**1.75**	**1.99**	**2.13**	**1.67**	**1.64**
	1.31	1.75	1.83	1.55	1.11	0.76

Table 4

Marginal Value Ratios between *Dollar Cost Averaging* and *Buy and Hold* purchase strategy payoffs for an investor who holds a 50:50 combination of the risk free asset and the market portfolio and commences purchases of randomly chosen securities in randomly chosen initial months between December 1925 and January 1998 and continues the purchases (DCA) or holds the securities (BH) for up to 72 months.

Securities are chosen either with *Equal Probability* (Panel A) or *Value-Weighted Probability* (Panel B). Figures in **bold** font relate to the **value-weighted** market portfolio, while figures in standard font relate to an **equally weighted** market portfolio. The Marginal Value Ratio is the ratio of the expected marginal utilities per dollar invested in the Dollar Cost Averaging and Buy and Hold strategies.

Panel A: Equal Probability Security Selection

	Horizon (months)					
Relative Risk Aversion	12	24	36	48	60	72
2	**0.97**	**0.93**	**0.90**	**0.87**	**0.86**	**0.85**
	0.98	0.95	0.92	0.89	0.87	0.85
3	**0.99**	**0.96**	**0.95**	**0.93**	**0.93**	**0.91**
	1.00	0.98	0.98	0.95	0.92	0.89
4	**1.00**	**1.00**	**1.02**	**1.02**	**1.02**	**0.99**
	1.02	1.03	1.06	1.03	0.98	0.94
5	**1.02**	**1.05**	**1.14**	**1.15**	**1.14**	**1.09**
	1.04	1.10	1.20	1.14	1.03	1.00
6	**1.04**	**1.14**	**1.32**	**1.32**	**1.28**	**1.22**
	1.06	1.20	1.41	1.31	1.09	1.05
7	**1.06**	**1.26**	**1.55**	**1.55**	**1.43**	**1.35**
	1.09	1.33	1.67	1.51	1.13	1.09

Panel B: Value Weighted Security Selection

	Horizon (months)					
Relative Risk Aversion	12	24	36	48	60	72
2	**0.98**	**0.96**	**0.94**	**0.92**	**0.91**	**0.89**
	0.98	0.96	0.94	0.91	0.88	0.86
3	**0.99**	**0.99**	**0.98**	**0.97**	**0.95**	**0.94**
	1.00	0.99	0.97	0.94	0.90	0.88
4	**1.00**	**1.02**	**1.04**	**1.03**	**1.01**	**1.00**
	1.01	1.03	1.03	0.98	0.93	0.90
5	**1.02**	**1.07**	**1.13**	**1.12**	**1.09**	**1.07**
	1.03	1.08	1.12	1.05	0.95	0.91
6	**1.04**	**1.14**	**1.26**	**1.25**	**1.19**	**1.16**
	1.04	1.15	1.25	1.14	0.98	0.92
7	**1.06**	**1.23**	**1.42**	**1.40**	**1.29**	**1.25**
	1.07	1.25	1.41	1.25	1.01	0.91

Table 5

Marginal Value Ratios between *Dollar Cost Averaging* and *Buy and Hold* purchase strategy payoffs for an investor who holds the optimal combination of the risk free asset and the market portfolio, and commences purchases of randomly chosen securities in randomly chosen initial months between December 1925 and January 1998 and continues the purchases (DCA) or holds the securities for up to 72 months.

Securities are chosen either with *Equal Probability* (Panel A) or *Value-Weighted Probability* (Panel B). Figures in **bold** font relate to the **value-weighted** market portfolio, while figures in standard font relate to an **equally weighted** market portfolio. $x^* \equiv (\bar{R}_M - R_f)/\gamma\sigma^2{}_M$ is the optimal allocation to the market portfolio for a myopic investor, where γ is the coefficient of relative risk aversion, σ_M is the sample volatility of the market portfolio, and $\bar{R}_M - R_f$ is the sample excess return on the market portfolio. The Marginal Value Ratio is the ratio of the expected marginal utilities per dollar invested in the Dollar Cost Averaging and Buy and Hold strategies.

Panel A: Equal Probability Security Selection

		Horizon (months)					
Relative Risk Aversion	x^*	12	24	36	48	60	72
2	**1.06**	**1.01**	**1.02**	**1.07**	**1.09**	**1.08**	**1.05**
	0.89	1.01	1.02	1.04	1.02	0.98	0.95
3	**0.71**	**1.00**	**1.01**	**1.05**	**1.05**	**1.05**	**1.02**
	0.59	1.01	1.01	1.03	1.00	0.96	0.93
4	**0.53**	**1.00**	**1.01**	**1.05**	**1.04**	**1.04**	**1.01**
	0.45	1.01	1.01	1.02	0.99	0.95	0.91
5	**0.42**	**1.00**	**1.01**	**1.04**	**1.04**	**1.04**	**1.01**
	0.36	1.01	1.01	1.02	0.98	0.94	0.90
6	**0.35**	**1.00**	**1.01**	**1.04**	**1.04**	**1.04**	**1.01**
	0.30	1.01	1.01	1.02	0.98	0.93	0.90
7	**0.30**	**1.00**	**1.01**	**1.05**	**1.04**	**1.04**	**1.01**
	0.25	1.01	1.01	1.01	0.97	0.93	0.89

Panel B: Value Weighted Security Selection

		Horizon (months)					
Relative Risk Aversion	x^*	12	24	36	48	60	72
2	**1.06**	**1.01**	**1.04**	**1.08**	**1.09**	**1.07**	**1.05**
	0.89	1.01	1.02	1.02	0.98	0.94	0.91
3	**0.71**	**1.01**	**1.03**	**1.06**	**1.06**	**1.04**	**1.02**
	0.59	1.00	1.01	1.00	0.97	0.92	0.89
4	**0.53**	**1.01**	**1.03**	**1.06**	**1.05**	**1.03**	**1.01**
	0.45	1.00	1.01	1.00	0.96	0.91	0.88
5	**0.42**	**1.01**	**1.03**	**1.05**	**1.05**	**1.03**	**1.01**
	0.36	1.00	1.01	1.00	0.95	0.91	0.88
6	**0.35**	**1.01**	**1.03**	**1.05**	**1.04**	**1.03**	**1.01**
	0.30	1.00	1.01	0.99	0.95	0.90	0.87
7	**0.30**	**1.01**	**1.03**	**1.05**	**1.04**	**1.03**	**1.01**
	0.25	1.00	1.01	0.99	0.95	0.90	0.87

Table 6

Marginal Value Ratios between *Dollar Cost Averaging* and *Buy and Hold* purchase strategy payoffs for an investor who holds the optimal combination of the risk free asset and the market portfolio for simulated market and security returns. Security and Market Returns are simulated using equations (7),(8), and(9). The Marginal Value Ratio is the ratio of the expected marginal utilities per dollar invested in the Dollar Cost Averaging and Buy and Hold strategies. Ratios shown in boldface are for an investor who holds an optimally levered position in the market portfolio; ratios shown in standard type are for an investor who holds an optimally levered position in a portfolio with a β of unity and the same expected return as the market portfolio, but with a residual risk of 5% per year. The parameter values used for Panel C are the base case parameters excepts as noted.

Panel A: Base Case Parameter Values

		$R_f = 0.037$	$E[R_M] = 0.114$	$\sigma_M = 0.191$	$\sigma_z = 0.05$
$\beta = 1.0$	$\sigma_\epsilon = 0.05$	$\sigma_{z_u} = 0.69$	$\rho = 0.85$	$\eta_1 = 0.15$	$\eta_2 = 0.05$
			$\gamma = 2$		

Panel B: Base Case Autocorrelations

	Lag (months)											
	1	2	3	4	5	6	7	8	9	10	11	12
Autocorrelation	0.06	-0.04	-0.07	-0.06	-0.05	-0.05	-0.04	-0.03	-0.03	-0.03	-0.02	-0.02

Panel C: Marginal Value Ratios

		Horizon (months)					
Model	Parameters	12	24	36	48	60	72
1.	Base	**0.999**	**1.035**	**1.061**	**1.076**	**1.074**	**1.081**
		0.997	1.030	1.053	1.065	1.061	1.066
2.	$\sigma_u = 0.0$	**1.001**	**1.000**	**1.001**	**1.003**	**0.997**	**1.000**
		0.999	0.996	0.994	0.994	0.985	0.985
3.	$\beta = 0.0$	**0.000**	**1.037**	**1.060**	**1.071**	**1.073**	**1.080**
		1.000	1.037	1.059	1.070	1.073	1.081
4.	$\rho = 0.900$	**0.976**	**1.020**	**1.057**	**1.076**	**1.091**	**1.103**
		0.974	1.015	1.050	1.067	1.077	1.087
5.	$\eta_1 = 0.25$	**0.990**	**1.044**	**1.068**	**1.080**	**1.085**	**1.100**
		0.988	1.039	1.060	1.070	1.071	1.083
6.	$\eta_2 = 0.150$	**0.997**	**1.039**	**1.060**	**1.074**	**1.088**	**1.089**
		0.995	1.035	1.053	1.065	1.074	1.074